

Exotic Hadrons in the Constituent Quark Model

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Exotic hadrons are important because their existence or absence can provide important clues to understanding how QCD makes hadrons from quarks and gluons. The first experimentally confirmed exotic will be the first hadron containing both qq and $\bar{q}q$ pairs and the first hadron containing color sextet and color octet pairs. Theoretical models are not very useful because there is no accepted model for multiquark systems with color-space correlations. The constituent quark model is the only phenomenological model with predictive power that has given experimentally tested universal predictions for both mesons and baryons. This paper reviews its explanation for why there are no bound exotics and its guidance to the search for heavy-flavored exotic tetraquarks and pentaquarks. A possible supersymmetry between mesons and baryons leading to meon-baryon mass relations not easily obtained otherwise is discussed.

§1. Why exotics and the constituent quark model are important

1.1. *Absence of exotics - an early clue to QCD*

The “Goldhaber Gap” (no K^+N resonances) led Gell-Mann to quarks. The experimental hadron spectrum today still shows no exotic bound states with exotic quantum numbers. The only bound states are color singlet ($q\bar{q}$) mesons and baryons containing color antitriplet (qq) pairs. Finding the first exotic would help our understanding how QCD makes hadrons from quarks and gluons. It would be the first multiquark hadron containing both q and \bar{q} and the first hadron containing both color sextet qq pairs and color octet ($\bar{q}q$) pairs.

Some crucial questions

1. How does QCD make hadrons from quarks and gluons?
2. DOES QCD make hadrons from quarks and gluons?
3. Do we need more than the standard model?
4. Do we need another symmetry or supersymmetry?
5. Why are there no bound exotics?
6. How can a pion be both a Goldstone Boson and $2/3$ of a nucleon?

1.2. *The constituent quark model helps understanding how QCD makes hadrons*

Experimental data show us mesons and baryons made of same quarks with flavor-independent interactions for five flavors. The constituent quark model is the only hadron model with demonstrated reliable predictive power, includes five-flavor meson-baryon universality and can treat states containing both qq and $\bar{q}q$. The model makes testable experimental predictions for the two-body interactions $V(q\bar{q})_8$ and $V(qq)_6$ and for states containing both qq and $\bar{q}q$. No other model makes such predictions where no experimental data are yet available and which are tested by the presence or absence of exotics.

§2. What are constituent quarks - A BCS Approach from John Bardeen

Quarks are quasiparticle degrees of freedom describing low-lying elementary excitations of hadronic matter. BCS theory was not gauge invariant - so what? Bardeen knew it had the right physics. Anderson explained the broken gauge invariance - found the Higgs mechanism. Particle physicists don't recognize that Anderson found Higgs. In 1960 Bardeen noted Nambu's interesting application of superconductivity ideas to particle physics. But at the 1960 International High energy (Rochester) Conference particle physicists showed no interest in Nambu's work on symmetry breaking. BCS theory must be obtainable from the QED Lagrangian, but nobody knew how to do it. Summing Feynman diagrams or putting electrons on a lattice still cannot lead to BCS.

The constituent quark model is not relativistic - so what? The experimental data show it has the right physics. Getting constituent quarks from QCD or the lattice might be as difficult as getting BCS from QED.

That meson and baryon masses are related because they are made of the same quarks was first pointed out by Sakharov and Zeldovich¹⁾ and completely independently rediscovered.^{2),3)} They noted that Λ and Σ were made of same quarks, *anticipated* QCD and explained their mass difference with a flavor-dependent hyperfine interaction. Their constituent quark mass formula gave two surprising¹⁾⁻³⁾ meson - baryon mass relations which were later followed by a number of additional successful quark model relations^{4),5)} with no simple explanations from QCD.

$$M = \sum_i m_i + \sum_{i>j} \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{m_i \cdot m_j} \cdot v_{ij}^{hyp} \quad (2.1)$$

These quasiparticle excitation of the QCD vacuum carry the same spin and flavor quantum numbers as current quarks. Mesons are $q\bar{q}$ pairs, baryons are $3q$ and nothing else. Their effective quark masses include all interaction energies except the color hyperfine energy and have the same values for mesons and baryons. Their spin-dependent interactions are given by effective moments equal to Dirac moments with the same effective quark mass. Hadron magnetic moments are the vector sums of quark moments. Hyperfine splittings are proportional to products of the same (color) moments.

The experimental successes of model - challenges for QCD - include three mag-

netic moment predictions with no free parameters⁴⁾

$$-1.46 = \frac{\mu_p}{\mu_n} = -\frac{3}{2}; \quad -0.61 \text{ n.m.} = \mu_\Lambda = -\frac{\mu_p}{3} \frac{M_{\Sigma^*} - M_\Sigma}{M_\Delta - M_N} = -0.61 \text{ n.m.} \quad (2.2)$$

$$\mu_p + \mu_n = 0.88 \text{ n.m.} = \frac{2M_p}{M_N + M_\Delta} = 0.865 \text{ n.m.} \quad (2.3)$$

The last prediction sets absolute scale of magnetic moments.

QCD calculations have not yet explained such remarkably successful simple constituent quark model results. It is not a nonrelativistic nor potential quark model. The internal structures and space-time properties of constituent quarks are not defined. What they really are remain as challenges for QCD.

A completely different experimental confirmation of this picture is seen in the successful relations between meson and baryon total cross sections;^{6),7)} e.g. the predictions for total cross sections at $P_{lab} = 100 \text{ GeV}/c$,

$$38.5 \pm 0.04 \text{ mb} = \sigma_{tot}(pp) = 3\sigma_{tot}(\pi^+p) - \frac{3}{2}\sigma_{tot}(K^-p) = 39.3 \pm 0.2 \text{ mb} \quad (2.4)$$

$$33.1 \pm 0.31 \text{ mb} = \sigma_{tot}(\Sigma p) = \frac{3}{2}\{\sigma_{tot}(K^+p) + \sigma_{tot}(\pi^-p) - \sigma_{tot}(K^-p)\} = 33.6 \pm 0.16 \text{ mb} \quad (2.5)$$

$$29.2 \pm 0.29 \text{ mb} = \sigma_{tot}(\Xi p) = \frac{3}{2}\sigma_{tot}(K^+p) = 28.4 \pm 0.1 \text{ mb} \quad (2.6)$$

But we still do not know what the constituent quark is.

§3. The LS transformation - A new meson-baryon supersymmetry?

3.1. The prediction for the newly discovered Σ_b baryons

A new challenge demanding explanation from QCD is posed by the remarkable agreement between the experimental masses⁸⁾ of the newly discovered Σ_b^+ and Σ_b^- and the prediction^{4),9)} from meson masses,

$$\frac{M_{\Sigma_b} - M_{\Lambda_b}}{(M_\rho - M_\pi) - (M_{B^*} - M_B)} = \frac{M_{\Sigma_c} - M_{\Lambda_c}}{(M_\rho - M_\pi) - (M_{D^*} - M_D)} = \frac{M_\Sigma - M_\Lambda}{(M_\rho - M_\pi) - (M_{K^*} - M_K)} \approx \frac{0.32}{0.33} \approx \frac{0.325}{0.325} \quad (3.1)$$

New successful relations, (3.1) and others described below indicate some light antiquark-diquark supersymmetry⁹⁾ between meson and baryon states not simply described by QCD treatments, which treat meson and baryon structures very differently. This light quark supersymmetry transformation, denoted here by T_{LS}^S , connects a meson denoted by $\mathcal{M}(\bar{q}Q_i)$ and a baryon denoted by $B([qq]_S Q_i)$ both containing the same valence quark of some fixed flavor Q_i , $i = (u, s, c, b)$ and a light color-antitriplet “brown muck” state with the flavor and baryon quantum numbers respectively of an antiquark \bar{q} (u or d) and two light quarks coupled to a diquark of spin S . No model is assumed for the valence quark nor for the brown muck antitriplet which is coupled to it. This goes beyond the simple constituent quark model and

holds also for the quark-parton model in which the valence is carried by a current quark and the rest of the hadron is a complicated mixture of quarks and antiquarks.

$$T_{LS}^S \cdot \mathcal{M}(\bar{q}Q_i) \equiv \mathcal{B}([qq]_S Q_i) \quad (3.2)$$

The mass difference between the meson and baryon related by this T_{LS}^S transformation has been shown⁹⁾ to be independent of the quark flavor i for all four flavors (u, s, c, b) for the two cases of spin-zero⁵⁾ $S = 0$ and spin-one⁹⁾ $S = 1$ diquarks, when we use weighted averages of hadron masses which cancel their hyperfine contributions,

$$\tilde{M}(V_i) \equiv \frac{3M_{V_i} + M_{P_i}}{4}; \quad \tilde{M}(\Sigma_i) \equiv \frac{2M_{\Sigma_i^*} + M_{\Sigma_i}}{3}; \quad \tilde{M}(\Delta) \equiv \frac{2M_{\Delta} + M_N}{3} \quad (3.3)$$

$$\begin{aligned} M(N) - \tilde{M}(\rho) &= M(\Lambda) - \tilde{M}(K^*) &= M(\Lambda_c) - \tilde{M}(D^*) &= M(\Lambda_b) - \tilde{M}(B^*) \\ 323 \text{ MeV} &\approx 321 \text{ MeV} &\approx 312 \text{ MeV} &\approx 310 \text{ MeV} \end{aligned} \quad (3.4)$$

$$\begin{aligned} \tilde{M}(\Delta) - \tilde{M}(\rho) &= M(\Sigma) - \tilde{M}(K^*) &= \tilde{M}(\Sigma_c) - \tilde{M}(D^*) &= \tilde{M}(\Sigma_b) - \tilde{M}(B^*) \\ 517.56 \text{ MeV} &\approx 526.43 \text{ MeV} &\approx 523.95 \text{ MeV} &\approx 512.45 \text{ MeV} \end{aligned} \quad (3.5)$$

The ratio of the hyperfine splittings of mesons and baryons related by T_{LS}^1 is also independent of the quark^{5), 10)} flavor i for all four flavors (u, s, c, b),

$$\begin{aligned} \frac{M_\rho - M_\pi}{M_\Delta - M_N} &= \frac{M_{K^*} - M_K}{M_{\Sigma^*} - M_\Sigma} &= \frac{M_{D^*} - M_D}{M_{\Sigma_c^*} - M_{\Sigma_c}} &= \frac{M_{B^*} - M_B}{M_{\Sigma_b^*} - M_{\Sigma_b}} \\ 2.17 \pm 0.01 &= 2.08 \pm 0.01 &= 2.18 \pm 0.01 &= 2.15 \pm 0.20 \end{aligned} \quad (3.6)$$

That masses of boson and fermion states containing quarks of four different flavors, u, d, s, b , related by this transformation (3.2) satisfy simple relations like (3.1), (3.4), (3.5) and (3.6) with no free parameters can hardly be accidental. They remain a challenge for QCD perhaps indicating some boson-fermion or antiquark-diquark supersymmetry. Any model for hadron spectroscopy which treats mesons and baryons differently or does not yield agreement with data for all five flavors is missing essential physics.

§4. The Nambu color exchange interaction (1966) anticipating QCD

4.1. Nambu's Theorem (1966) : No exotics. Only lowest color singlets are stable

Nambu¹¹⁾ considered colored quarks interacting via a non-abelian SU(3) gauge field in the theory now called QCD. The color-exchange interaction (one gluon exchange) between two constituents i and j is for a multi-quark system,

$$V_{cx}(r_{ij}) = \frac{V}{2} \cdot \sum_{i \neq j} \vec{\lambda}_c^i \cdot \vec{\lambda}_c^j \cdot v(r_{ij}) \quad (4.1)$$

where i and j can be either quarks or antiquarks, $v(r_{ij})$ depends on the space and spin variables of the constituents but is the same for all pairs, independent of i and j , $\vec{\lambda}_c^i$ is the color SU(3) generator and $\vec{\lambda}_c^i \cdot \vec{\lambda}_c^j$ denotes the scalar product in color space. In lowest order neglecting color-space and color-spin correlations

$$\frac{\langle V_{cx}(tot) \rangle}{\langle v(r) \rangle} = \frac{V}{2} \cdot \left[\sum_{ij} \vec{\lambda}_c^i \cdot \vec{\lambda}_c^j - \sum_i (\lambda_c^i)^2 \right] = \frac{V}{2} \cdot \left[(\lambda_C)^2 - \sum_i (\lambda_c^i)^2 \right] \quad (4.2)$$

where λ_C is the generator of the color SU(3) group for the whole multiquark system. The lowest states having N constituents are color singlet states having $\lambda_C = 0$. All states with the same overall color have the same potential energy.

Nambu's theorem predicts that the lowest bound hadrons are color singlet $(q\bar{q})_1$ mesons and color singlet baryons in which each quark pair is coupled to a color antitriplet $(qq)_{\bar{3}}$. Other color singlets are not bound as they can gain kinetic energy by breaking up into color singlets.

4.2. Color-space correlations and tetraquarks with the Nambu interaction

The tetraquarks are the simplest multiquark color singlet system. They exhibit features missed in other models suggested for these states and provide useful models for mesons containing heavy quarks; e.g. color sextet quark-quark couplings.

The first treatment of states containing both qq and $\bar{q}\bar{q}$ found new forces between color singlet hadrons in a tetraquark¹²⁾ $qq\bar{q}\bar{q}$ with the Nambu interaction (4.1). There are two color singlet couplings for $qq\bar{q}\bar{q}$ system. The qq can be coupled either to a color $\bar{3}$ or color $\bar{6}$ and combined respectively with the $\bar{q}\bar{q}$ in color 3 or color $\bar{6}$ to make a color singlet. The Nambu interaction gives the same energy for all color couplings in systems with maximum space symmetry.

The color sextet diquark is not present in normal baryons. The Nambu interaction gives a repulsive qq interaction exactly compensated by $q\bar{q}$ attraction in the $6\bar{6}$ configuration. Potential energy can be gained in $6\bar{6}$ by breaking space symmetry and keeping qq spacing larger than $q\bar{q}$. The model¹²⁾ showed this gain was not sufficient to overcome kinetic energy with equal quark masses and reasonable potentials.

Explicit calculations using a harmonic oscillator spatial dependence of the operator V in the interaction (4.2) with broken space symmetry¹³⁾ show that the (SS) state is always considerably below the $(\bar{3}3)$ state. For sufficiently unequal quark masses the (SS) is also below the two-meson threshold.

Calculations for the $(us\bar{d}\bar{c})$ system with four different flavors and four different masses show how the difference between the tetraquark mass and the mass of two separated mesons depends upon the quark masses.

When all the spatial separations are equal, $r_{us}^2 = r_{\bar{c}\bar{d}}^2 = r_{u\bar{c}}^2 = r_{s\bar{c}}^2 = r_{u\bar{d}}^2 = r_{s\bar{d}}^2$ the $q\bar{q}$ interaction in the state SS is 25% stronger than the $q\bar{q}$ interaction in the separated two-meson state $2M$. This additional attraction is balanced exactly by the us and $\bar{c}\bar{d}$ repulsions as required by Nambu's theorem. But more additional attraction is obtainable by breaking space symmetry and making the mean us and $\bar{c}\bar{d}$ distances larger than the mean $q\bar{q}$ distance.

The ratios of ground state energies for the $\bar{3}3$ and SS systems to the energy of

the two meson state are obtained by setting $m_u = m_d$ and substitute constituent quark masses obtained by fitting ground state meson and baryon spectra⁴⁾

$$m_u = 360 \text{ MeV}; m_s = 540 \text{ MeV}; m_c = 1710 \text{ MeV}; m_b = 5050 \text{ MeV} \quad (4.3)$$

$$\frac{E_g(\mathbf{\bar{3}\bar{3} usdc})}{E_g(2M usdc)} = 1.18; \quad \frac{E_g(SS usdc)}{E_g(2M usdc)} = 1.11 \quad (4.4)$$

The (SS) state is considerably below the $(\mathbf{\bar{3}\bar{3}})$ state

When heavier quark states are included the ratio of the (SS) mass to the two-meson threshold is seen to drop with increasing quark mass and is actually below the threshold for the *cubu* and *bubu* tetraquarks.

$$\frac{E_g(SS subu)}{E_g(2M subu)} = 1.077; \quad \frac{E_g(SS cucu)}{E_g(2M cucu)} = 1.036; \quad \frac{E_g(SS cubu)}{E_g(2M cubu)} = 0.975; \quad \frac{E_g(SS bubu)}{E_g(2M bubu)} = 0.891 \quad (4.5)$$

In this approximation the mass of the $cu\bar{c}\bar{u}$ tetraquark is only 4% above the $D\bar{D}$ threshold and the $bu\bar{b}\bar{u}$ tetraquark is well below the $B\bar{B}$ threshold. Such low-mass thresholds may well be found in a more exact calculation including spin effects. That they might be found in the experimental spectrum must be taken seriously. The color-space correlation contributions to the energy may well be more important than the color-magnetic energy commonly used in model calculations.

This casts doubt on tetraquark calculations neglecting color space correlations; e.g. those for the $X(3872)$ resonance. QCD tells us space symmetry is broken by the strongly attractive $q\bar{q}_1$ interaction, much stronger than the attractive qq_3 interaction.

4.3. Color-spin correlations with Nambu's interaction

The DeRujula-Georgi-Glashow Model¹⁴⁾ introduced a color-spin hyperfine interaction and obtained remarkable agreement with nonexotic hadron spectra, including masses, spin splittings and magnetic moments. Jaffe¹⁵⁾ extended the DGG model¹⁴⁾ to treat multiquark systems using the Nambu interaction (4.1) to predict $V(q\bar{q})_8/V(q\bar{q})_1$ and $V(qq)_6/V(qq)_{3*}$ which are not obtainable from the nonexotic hadron spectrum. The Jaffe model explains striking features of the hadron spectrum. Pauli principle "flavor antisymmetry" requires repulsive short-range color-magnetic interaction between same-flavor quark pairs thus explaining the absence of low-lying bound exotics like a dipion with a mass less than two pion masses or a dibaryon bound by 100 MeV. Only the short-range color-magnetic interaction can produce multiquark binding. Best candidates with a minimum number of repulsive same-flavor pairs are Jaffe's H ($uuddss$) dibaryon and the anticharmed strange ($uuds\bar{c}$) pentaquark. The Θ^+ pentaquark ($uudd\bar{s}$) has too many same-flavor quark pairs and was not considered.

§5. Guide to searches for exotics

5.1. The $\Theta^+(uudd\bar{s})$ pentaquark

The Θ^+ is discussed extensively elsewhere.¹⁶⁾ A color-space correlation similar to those for our tetraquarks is found in one calculation¹⁷⁾ using a harmonic oscillator

hamiltonian with an additional spin-dependent interaction in a model space of 15,000 basic single-particle shell-model wave functions. The mean quark-antiquark distance is much smaller than the mean quark-quark distance. The one-body r.m.s. radius measured from the center of mass is 1.10 Fm. for u, d quarks and 0.72 Fm for the \bar{s} antiquark while the corresponding radius is 0.69 Fm for the $(0s)^5$ configuration.

That the one-body r.m.s. radius of the multiquark system is larger than expected from normal hadrons implies a short-range color-magnetic interaction weaker than in normal hadrons. The conventional practice of using spin splittings from normal hadrons to normalize the color-magnetic interaction^{4), 15)} is therefore questionable.

5.2. How to search for stable exotic baryons

Ashery at E791 searched for the $\bar{c}uuds$ pentaquark,¹⁸⁾ found events, but not enough to be convincing. The possible existence of this pentaquark is still open. Better searches are needed. A simple clean search for protons from a secondary vertex would see the signature for weakly decaying baryon and would immediately find all new weakly decaying baryons. Jaffe model calculations found no reason to look for the $\Theta^+(uudd\bar{s})$ pentaquark.

5.3. Experimental candidates for tetraquark searches

The best candidates for experimental detection are the $c\bar{u}b\bar{u}$ and $s\bar{u}b\bar{u}$ tetraquarks which cannot decay strongly or electromagnetically into light quark mesons.

A $bq\bar{c}\bar{q}$ tetraquark with isospin 1 and a mass below the $B\bar{D}$ threshold but above the mass of the $B_c\pi$ system can decay strongly into a $B_c\pi$ with a width limited by phase space. Both isospin states below the $B_c\pi$ threshold should be narrow and decay electromagnetically into $B_c\gamma$. A spin-zero state must decay by e^+e^- emission. An $I = 0$ state above $B_c\pi$ threshold can decay into $B_c\pi$ via isospin violation. These considerations apply also to states below the $B^*\bar{D}$ threshold that cannot decay into $B\bar{D}$. A $bq\bar{c}\bar{q}$ tetraquark below the B_c mass can decay only weakly and can appear in invariant mass plots of final states like $J/\psi e\nu$ as an additional mass peak along with the B_c . Isovector tetraquarks like $bu\bar{c}\bar{d}$ or $bd\bar{c}\bar{u}$ have exotic final states with wrong charges, like $J/\psi\eta$ or $J/\psi\pi^-\pi^-$. Searches for such tetraquarks are of interest in experiments observing the B_c . One can look for monoenergetic photons or pions emitted together with a B_c , a doublet structure of the B_c mass and exotic B_c decays. Analogous considerations hold for $bq\bar{s}\bar{q}$ tetraquarks with masses below BK threshold.

Exotic multiquark states with color-space correlations have a larger extension in space than normal hadrons, Production of such states make be difficult as they can be easily broken up by final state interactions or rescattering.

§6. Conclusions

The Nambu interaction explains the absence of strongly bound exotics and provides guide lines for future searches. Color-space correlations may be more important than color-flavor-spin correlations dominating other treatments. Basic QCD physics implies the $q\bar{q}$ interaction observed in mesons is much stronger in multiquark systems than the qq interaction observed in baryons, produces admixtures of quark

states absent in normal baryons^{4),16)} and destroys all diquark structures. Color-space correlated tetraquarks may be found in mesons containing heavy quarks.

Searches for tetraquarks in the B_c system may find $bq\bar{c}\bar{q}$ tetraquarks below the $B\bar{D}$ threshold. Possible exotic signatures include strong or electromagnetic decays into a πB_c or γB_c , weak decays producing additional peaks in the mass spectrum of B_c decay final states or final states with exotic electric charge, like $J/\psi\eta$ or $J/\psi\pi^-\pi^-$.

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